THE t TESTS OF SIGNIFICANCE

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What is a t –test?

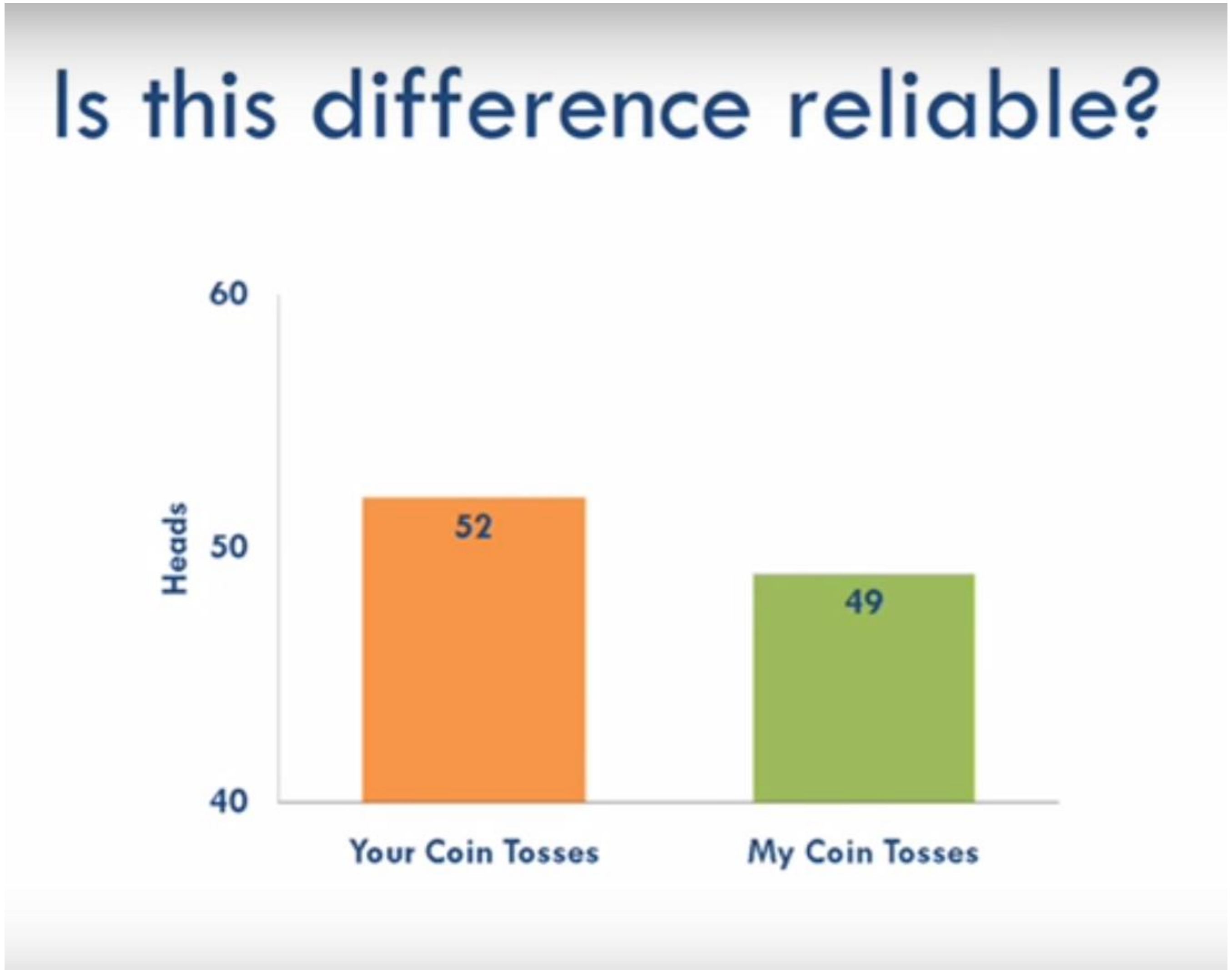
A t-test is a statistics that checks if two means (average) are reliably different from each other

It is a statistical test that allows the significance of difference between the means of the two samples to be determined

Why not just look at the means??

- Looking at means may show a difference, but we can’t be sure if that is a ***reliable*** difference

Exam: Coin flipping



This shows the difference between Descriptive Statistics and Inferential Statistics

Descriptive Statistics

These are stats, such as a mean, that describe data you have, but can't be generalized beyond that.

Inferential Statistics

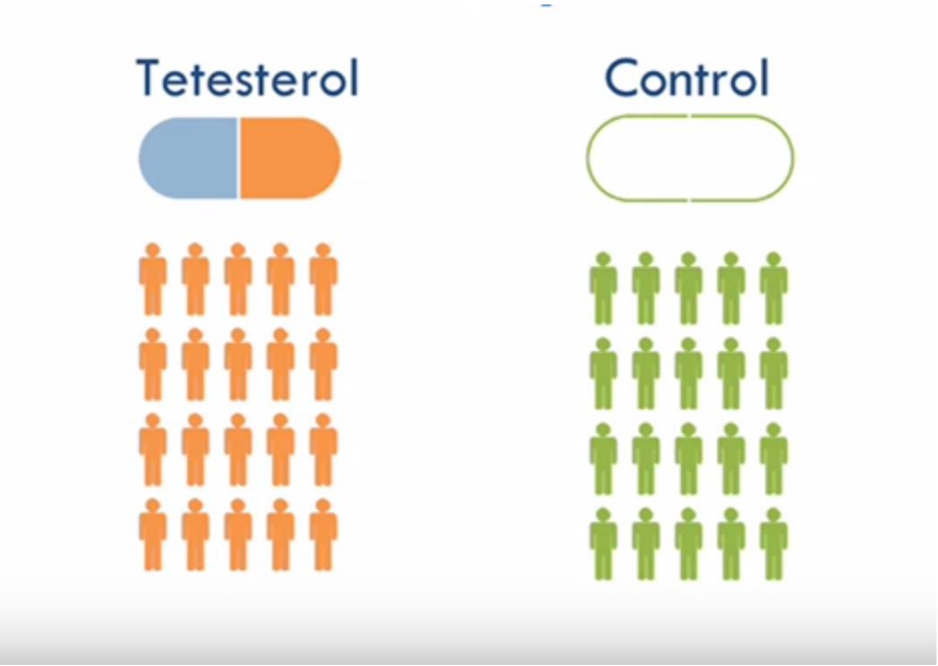
These are stats, such as t-tests, that allow us to make inferences about the population beyond our data.

Exam.. Drug testing

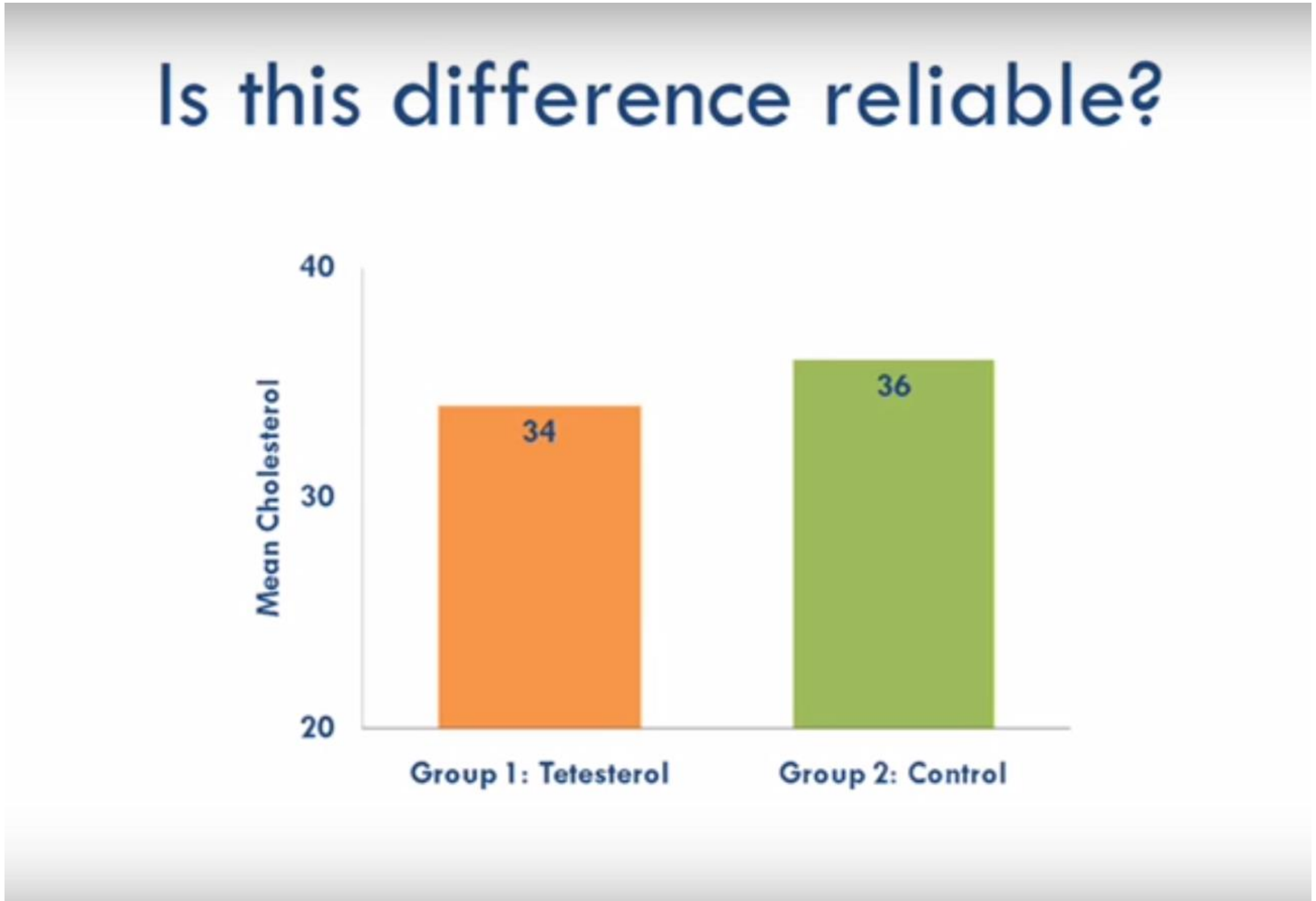
Researchers have developed a new drug that they hope will lowers cholesterol..

2 groups

- Treatment group
- Control group



After one month…



How do we know if the t-value is big enough to show a difference?

Each t-value has a p-value.

The p-value tells us the likelihood that there is a real difference.

The p-value

Specifically, the p-value is the probability that the pattern of data in the sample could be produced by random data.

The p-value

If p = .10, there is a 10% chance.

If p = .05, there is a 5% chance
there is no real difference.

If p = .01, there is a 1% chance.

Sample size

The p-value for each t-value also depends on the sample size.

Bigger samples make it easier to detect differences.

Sample size

With two groups of 5,
when $t = 2.0$, $p = .04$.

With two groups of 10,
when $t = 2.0$, $p = .03$.

Sample size

Bigger samples help, but with diminishing returns.

A good guideline is to aim for 20 to 30+ datapoints in each group.

"Degrees of Freedom" (df) is equal to sample size minus one.

Independent-samples t-test

Tests the means of two different groups.

Example: Testing the average quality of two different batches of beer.

Independent-samples t-test

Tests the means of two different groups.

Example: Testing the average quality of two different batches of beer.

Also called: between-samples and unpaired-samples t-test.

Paired-samples t-test

Tests the mean of one group twice.

Example: Testing balance before and after drinking.

Also called: within-subjects, repeated-measures and dependent-samples.

One-sample t-test

Tests the mean of one group against a set mean.

One-sample t-test

Tests the mean of one group against a set mean.

Example: Testing if your co-workers' IQs differ from the average of 100.

T-Test: Drawing Conclusions

1. Set Null Hypothesis (H_0): There **is no** significant difference
2. Set the critical P level at $P = 0.05$ (5%), which means 95% confident (accepted level in biology)

3. Write the decision rule for rejecting the null hypothesis:
   a. If calculated t value < critical t value, then accept null hypothesis (H_0)
   b. If calculated t value > critical t value, then reject null hypothesis (H_0)

T-Test: Drawing Conclusions

4. Determine the Degrees of Freedom Number (Total Sample Size - 2) and Critical Value using Chart

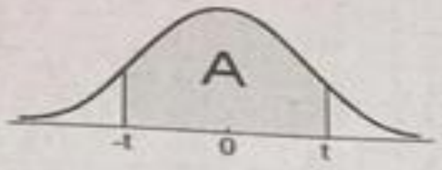
5. Write summary statement based on decision
   a. The null hypothesis is rejected because...
      i. ex: t value (2.28) > critical value (1.73)

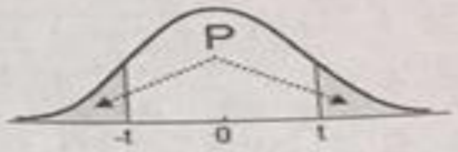
6. Write statements of results that includes hypothesis

T-Test: Drawing Conclusions

- Calculate T Value for pair of datasets and compare to Critical Values
- Use the **95% confidence level, called P Value (P)**

Critical Values of t
T-Distribution Table
(two tailed)





DF	A / P	0.80 / 0.20	0.90 / 0.10	0.95 / 0.05	0.98 / 0.02	0.99 / 0.01	0.995 / 0.005	0.998 / 0.002	0.999 / 0.001
1		3.078	6.314	12.706	31.820	63.657	127.321	318.309	636.619
2		1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3		1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4		1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5		1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6		1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7		1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8		1.397	1.860	2.306	2.897	3.355	3.833	4.501	5.041
9		1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10		1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11		1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12		1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13		1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14		1.345	1.761	2.145	2.625	2.977	3.326	3.787	4.140
15		1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16		1.337	1.746	2.120	2.584	2.921	3.252	3.686	4.015
17		1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18		1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19		1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20		1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21		1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
[illegible]		[illegible]	1.717	2.074	2.508	2.819	3.119	3.505	3.792

Limitations

1. The results of inferential statistics can only be applied to populations that resemble the sample that was tested.

Limitations

2. Your sample and population should be roughly normal in distribution. This means most scores will be around the mean with fewer scores further out, resembling a bell curve.

Limitations

3. Each group should have about the same number of datapoints. Comparing large and small groups together may give inaccurate results.

Limitations

4. All data should be independent. This means the scores should not be influenced by each other.

Limitations

5. Your data should be approximately interval-level or higher. This means each unit of measurement should be about equal to any other unit.

Problem Solving using the types of t-Test

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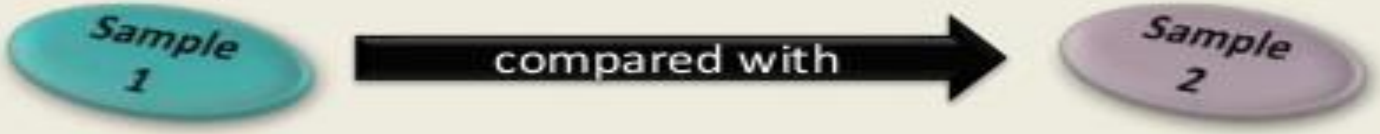
Differentiating Parametric Tests that Capture Differences Between Groups

T-test Family

- *Single sample t-test* – compares a single sample with its supposed population

- *Independent* samples t-test – compares one sample with another sample



- *Paired-samples t-test* – compares one sample on "time one" with itself on "time two"

NEXT

Comparison of MEANS	Degrees of Freedom	Application	Assumptions	Test Statistic
One Sample Z-Test	Not Applicable	Testing the difference of a sample mean, x-bar, with a known population mean, μ (fixed mean, historical mean, or targeted mean)	Normal distribution Known population σ.	$Z = \dfrac{\bar{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$
One Sample t-test	n-1	Testing the difference of one sample mean, x-bar, with a known population mean, μ (fixed mean, historical mean, or targeted mean)	Normal distribution Population standard deviation, σ, is unknown.	$t = \dfrac{\bar{x} - \mu}{\dfrac{s}{\sqrt{n}}}$
Two Sample t-test	$n_1 + n_2 - 2$	Testing difference of two sample means when population variances unknown but <u>considered equal</u>	Normal Distribution Requires standard pooled deviation calculation, s_p	$t = \dfrac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$
Paired t-test	n - 1	Testing two sample means when their respective population standard deviations are unknown but considered equal. Data recorded in pairs and each pair has a difference, d.	Normal Distribution Two dependent samples Always two-tailed test S_d = standard deviation of the differences of all samples	$t = \dfrac{\bar{d}\sqrt{n}}{s_d}$
One-Way ANOVA	$n_1 - 1$ & $n_2 - 1$	Testing the difference of three or more population means	Normal Distribution s_1^2 and s_2^2 represent sample variances	$F = \dfrac{(s_1)^2}{(s_2)^2}$

Hypothesis Testing

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Type Of Test	Purpose	Example	Equation	Comment	Excel Function
Z Test	Test if the average of a single population is equal to a target value	Do babies born at this hospital weigh more than the city average	$Z = \dfrac{\bar{x} - u_0}{\frac{\sigma}{\sqrt{n}}}$	Z test does not need df σ = population standard deviation	=Ztest(array,x,sigma)
1 Sample T-Test	Test if the average of a single population is equal to a target value	Is the average height of male college students greater than 6.0 feet?	$t = \dfrac{\bar{x} - u_0}{\frac{s}{\sqrt{n}}}$ $df = n - 1$	s = sample standard deviation	no built in equation use =STDEVA for standard deviation use =AVERAGE for mean use =T.DIST.RT to get 1 tailed confidence use =T.DIST.2T to get 2 tailed confidence
Paired T-Test	Test if the average of the differences between paired or dependent samples is equal to a target value	Weigh a set of people. Put them on a diet plan. Weigh them after. Is the average weight loss significant enough to conclude the diet works?	$t = \dfrac{\bar{d}}{\sqrt{\frac{s^2}{n}}}$ $df = n - 1$	d bar = average difference between samples s = sample deviation of the difference n = count of one set of the pairs (don't double count)	=TTEST(Array1,Array2,*,1) * -> 1 for 1 tailed, 2 for 2 tailed
2 Sample T-Test Equal Variance	Test if the difference between the averages of two independent populations is equal to a target value	Do cats eat more of type A food than type B food	$df = n_1 + n_2 - 2$ $t = \dfrac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} * \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$	n1, n2 = count of sample 1, 2	=TTEST(Array1,Array2,*,2)
2 Sample T-Test Unequal Variance	Test if the difference between the averages of two independent populations is equal to a target value	Is the average speed of cyclists during rush hour greater than the average speed of drivers	$t = \dfrac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$	$df = \dfrac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$	=TTEST(Array1,Array2,*,3)

Procedures in doing a t-test

1. Determine H_0 and H_1
2. Set the criterion
   - Look up t_{crit}, which depends on alpha and df
3. Collect sample data, calculate $\bar{x}$ and **s**
4. Calculate the test statistic

$$t_{obt} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

5. Reject H_0 if t_{obt} is more extreme than t_{crit}

t-table.pdf

Example:

A population of heights has a $\mu=68$. What is the probability of selecting a sample of size $n=25$ that has a mean of 70 or greater and a $s=4$?

- We hypothesized about a population of heights with a mean of 68 inches. However, we do not know the population standard deviation. This tells us we must use a t-test instead of a z-test

Step 1: State the hypotheses

H_0: $\mu=68$

H_1: $\mu\geq68$

Step 2: **Set the criterion**

- one-tail test or two-tail test?
- α=?
- df = n-1 = ?
- See table for critical t-value

Step 3: **Collect sample data, calculate** ***x*** **and s**

From the example we know the sample mean is 70, with a standard deviation (s) of 4.

t-table.pdf

Step 4: **Calculate the test statistic**

- Calculate the estimated standard error of the mean

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{4}{\sqrt{25}} = 0.8$$

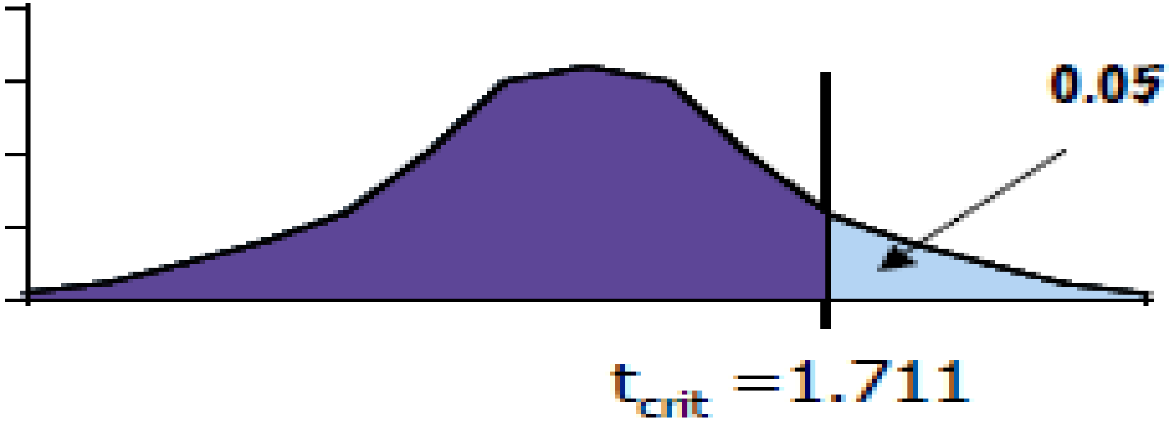
- Calculate the t-statistic for the sample

$$t = \frac{\bar{x} - \mu}{s_{\bar{x}}}$$

$$t = \frac{70 - 68}{0.8} = 2.5$$

<u>Step 5</u>: Reject H_0 if t_{obt} is more extreme than t_{crit}

- The critical value for a one-tailed t-test with df=24 and α=.05 is 1.711
- Will we reject or fail to reject the null hypothesis?



Hypothesis Tests for Two Population Means

T-TEST STATISTIC (EQUAL POPULATION VARIANCES)

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \qquad df = n_1 + n_2 - 2$$

where:

$\bar{x}_1$ *and* $\bar{x}_2$: **Sample means from populations 1 and 2**

$\mu_1 - \mu_2 =$ **Hypothesized difference**

n_1 *and* n_2 : **Sample sizes from the two populations**

$s_p =$ **Pooled standard deviation**

Example:

A researcher is interested in determining whether or not review sessions affect exam performance.

The independent variable, a review session, is administered to a sample of students (n=9) in an attempt to determine if this has an effect on the dependent variable, exam performance.

Based on information gathered in previous semesters, I know that the population mean for a given exam is 24.

The sample mean is 25, with a standard deviation (s) of 4.

- We hypothesized about a population mean for students who get a review based on the information from the population who didn't get a review (μ=24). However, we do not know the population standard deviation. This tells us we must use a t-test instead of a z-test

Step 1: State the hypotheses

H_0: $\mu=24$

H_1: $\mu \geq 24$

Step 2: Set the criterion

- one-tail test or two-tail test?
- α=?
- df = n-1 = ?
- See table for critical t-value

Step 3: Collect sample data, calculate *x* and s

From the example we know the sample mean is 25, with a standard deviation (s) of 4.

Step 4: **Calculate the test statistic**

- Calculate the estimated standard error of the mean

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{4}{\sqrt{9}} = \frac{4}{3} = 1.33$$

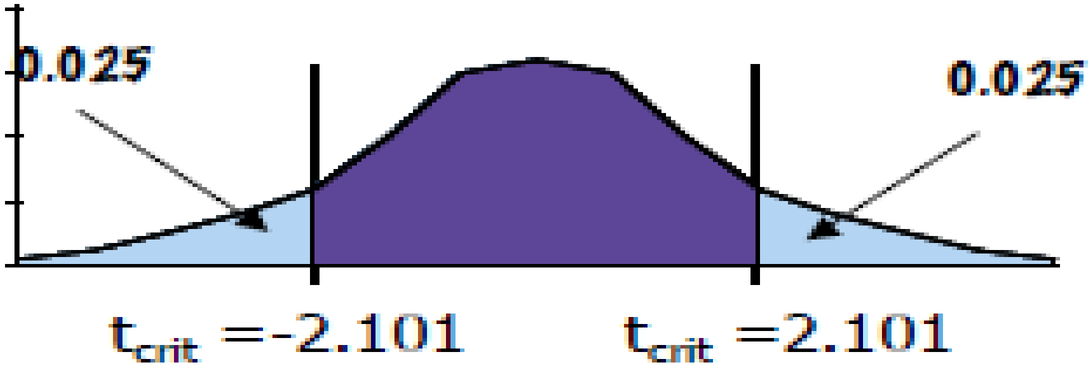
- Calculate the t-statistic for the sample

$$t = \frac{\bar{x} - \mu}{s_{\bar{x}}}$$

$$t = \frac{26 - 24}{1.33} = \frac{2}{1.33} = \underline{1.503}$$

<u>Step 5</u>: Reject H_0 if t_{obt} is more extreme than t_{crit}

- The critical value for a one-tailed t-test with df=8 and α=.05 is 1.86
- Will we reject or fail to reject the null hypothesis?



ONE SAMPLE T TEST

- A researcher believes that in recent years women have been getting taller. She knows that 10 years ago the average height of young adult women living in her city was 63 inches. The standard deviation is unknown. She randomly samples eight young adult women currently residing in her city and measures their heights. The following data obtained: 64,66,68,60,62,65,66,63

x	$x - \bar{x}$	$(x - \bar{x})^2$
64		
66		
68		
60		
62		
65		
66		
63		
N=8		

x	$x - \bar{x}$	$(x - \bar{x})^2$
64	-0.25	0.63
66	1.75	3.06
68	3.75	14.06
60	-4.25	18.06
62	-2.25	5.06
65	0.75	0.56
66	1.75	3.06
63	-1.25	1.56
N=8		$\Sigma(x - \bar{x})^2 =$ 46.05

SOLUTION

- Solving for the mean and variance of the sample we have x=64.25 and $s^{2=}$ 6.5
- Ho: u = 63 inches
- H1:u >63 inches (one tailed)
- For this particular problem we can set the level of significance to be at 0.05.
- Since n=8, we have df= n- 1=8-1=7. Since this is one tailed, we first multiple 2 to .05 level of significance, so a=0.10 and refer to appendix using df=7. Hence the critical vale of t=1.895
- Solve for the computed t
- Decision: Since the absolute value of t-computed=1.39 is less than the absolute value of t-critical =1.895 the we accept Ho. Hence, what the researcher thinks about the women getting taller in her city is not true.

T- test for Paired Samples

- A school teacher checks the curriculum content of a mathematics course subject to curriculum development. To do this random sample of 10 students was taken to check their knowledge growth from the beginning of the term of the mid semester undergoing the present curriculum. A diagnostic and a summative test were conducted at the beginning and the middle of the semester respectively. And here are the following :

DIAGNOSTIC TEST	SUMMATIVE TEST	d	d^2
79	81		
81	80		
83	80		
84	85		
84	80		
86	87		
86	86		
87	88		
88	86		
90	91		
		$\Sigma d=$	$\Sigma d^{2=}$

DIAGNOSTIC TEST	SUMMATIVE TEST	d	d^2
79	81	-2	4
81	80	1	1
83	80	3	9
84	85	-1	1
84	80	4	16
86	87	-1	1
86	86	0	0
87	88	-1	1
88	86	2	4
90	91	-1	1
		$\Sigma d=4$	$\Sigma d^2= 38$

SOLVE ☺ SOLVE ☺ ON THE BOARD

SOLUTION

- HYPOTHESIS
- LEVEL OF SIGNIFICANCE .05
- DEGREE OF FREEDOM, CRITICAL VALUE OF 1.833
- COMPUTE FOR t
- DECISION

T- TEST FOR UNPAIRED

- A study was conducted to compare the effectiveness of two teaching methods of statistics. The first method is via method A and the second is via method B. A group of 15 students were randomly chosen to undergo method A and group of 12 students were taken to to undergo method B. At the end of the term the same test was given to both groups and group A was found out to have an average of 86 with a standard deviation of 3, while the other group was find out to have an average of 84 with a standard deviation of 6. Assuming that the distribution were approximately normal and with 0.01 level of significance difference between method A and Method B?

X1=86	X2=84	N1=15
		N2=12

SOLUTION

- HYPOTHESIS
- Level of significance
- Degree of freedom with the critical value
- Computations
- Decisions

TRY AND SOLVE

- 1. A medical investigation claims that the average number of infections per week at a hospital in Southwestern Pennsylvania is 16.3 a random sample of 10 weeks had a mean number of 17.7 infections. The sample standard deviation is 1.8. If there enough evidence to reject the investigator’s claim at alpha =0.05? Assume the variable is normally distributed?

- 2. The average size of a farm in Indiana County, Pennsylvania is 191 acres. The average size of a farm in Green County is 199 acres. Assume the data were obtained from 2 samples with standard deviation of 38 and 12 acres, respectively, and sample sizes of 8 and 10, respectively. Can it be concluded at alpha = 0.05 that the average size of the farms in the counties is different? Assume the populations are normally distributed.


thank you!


WhatsCAP
Tank yu!
(Thank you!)